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11. RC circuits. Charging and discharging of capacitors

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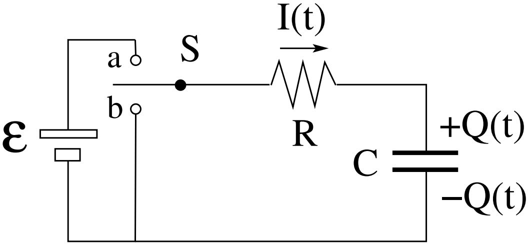
RC Circuit: Fundamentals

Specifications:

- $\mathcal{E}$ (emf)
- R (resistance)
- C (capacitance)

Switch S:

- a: charging
- b: discharging



Time-dependent quantities:

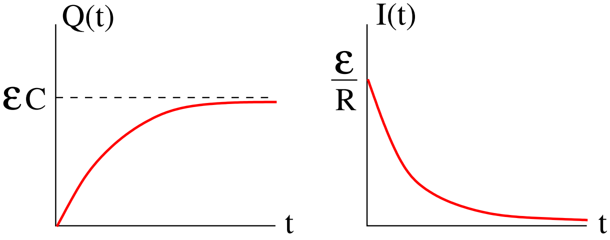
- $Q(t)$: instantaneous charge on capacitor
- $I(t) = \dfrac{dQ}{dt}$: instantaneous current
- $V_R(t) = I(t)R$: instantaneous voltage across resistor
- $V_C(t) = \dfrac{Q(t)}{C}$: instantaneous voltage across capacitor

RC Circuit: Charging the Capacitor

- Loop rule: $\mathcal{E} - IR - \dfrac{Q}{C} = 0$
- Differential equation: $R\dfrac{dQ}{dt} + \dfrac{Q}{C} = \mathcal{E} \quad \Rightarrow \dfrac{dQ}{dt} = \dfrac{\mathcal{E}C - Q}{RC}$

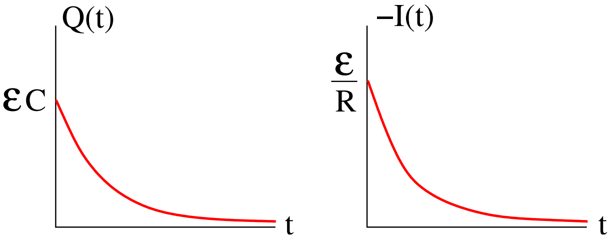
$$\int_0^Q \frac{dQ}{\mathcal{E}C - Q} = \int_0^t \frac{dt}{RC} \quad \Rightarrow \quad -\ln\left(\frac{\mathcal{E}C - Q}{\mathcal{E}C}\right) = \frac{t}{RC} \quad \Rightarrow \quad \frac{\mathcal{E}C - Q}{\mathcal{E}C} = e^{-t/RC}$$

- Charge on capacitor: $Q(t) = \mathcal{E}C\left[1 - e^{-t/RC}\right]$
- Current through resistor: $I(t) \equiv \dfrac{dQ}{dt} = \dfrac{\mathcal{E}}{R}\, e^{-t/RC}$



RC Circuit: Discharging the Capacitor

- Loop rule: $IR + \frac{Q}{C} = 0$
- Differential equation: $R\frac{dQ}{dt} + \frac{Q}{C} = 0 \quad \Rightarrow \frac{dQ}{dt} = -\frac{Q}{RC}$
  $$\Rightarrow \int_{\mathcal{E}C}^{Q} \frac{dQ}{Q} = -\int_0^t \frac{dt}{RC} \quad \Rightarrow \ln\left(\frac{Q}{\mathcal{E}C}\right) = -\frac{t}{RC} \quad \Rightarrow \frac{Q}{\mathcal{E}C} = e^{-t/RC}$$
- Charge on capacitor: $Q(t) = \mathcal{E}Ce^{-t/RC}$
- Current through resistor: $I(t) \equiv \frac{dQ}{dt} = -\frac{\mathcal{E}}{R}\, e^{-t/RC}$

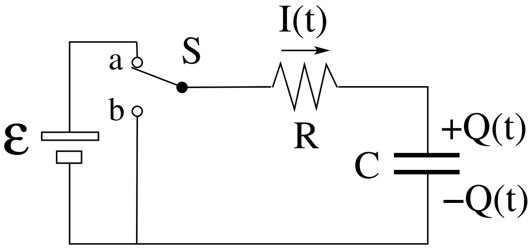


RC Circuit: Energy Transfer While Charging

Loop rule: $IR + \frac{Q}{C} = \mathcal{E}$ (I is positive)

- $I\mathcal{E}$: rate at which emf source delivers energy
- $IV_R = I^2R$: rate at which energy is dissipated in resistor
- $IV_C = \frac{IQ}{C}$: rate at which energy is stored in capacitor

Balance of energy transfer: $I^2R + \frac{IQ}{C} = I\mathcal{E}$

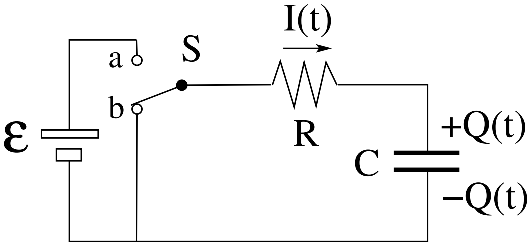


RC Circuit: Energy Transfer While Discharging

Loop rule: $IR + \frac{Q}{C} = 0$ (I is negative)

- $IV_R = I^2R$: rate at which energy is dissipated in resistor
- $IV_C = \frac{IQ}{C}$: rate at which capacitor releases energy

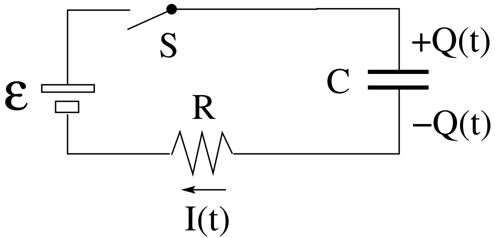
Balance of energy transfer: $I^2R + \frac{IQ}{C} = 0$



RC Circuit: Some Physical Properties

Specification of RC circuit by 3 device properties:

- $\mathcal{E}$ [V] (emf)
- R $[\Omega]$ (resistance)
- C [F] (capacitance)



Physical properties of RC circuit during charging process determined by 3 combinations of the device properties:

- $\mathcal{E}/R = I(t=0)$: rate at which charge flows onto capacitor initially
- $\mathcal{E}C = Q(t=\infty)$: total charge placed on capacitor ultimately
- $RC = \tau$: time it takes to place 63% of the charge onto the capacitor
  $[1 - e^{-1} = 0.632\ldots]$

At time $t = 0$ the capacitor in this circuit is discharged and the switch is being closed.

Find the current I_1

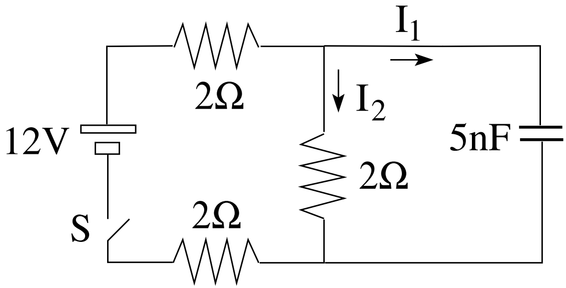
(a) at $t = 0$,

(b) at $t = \infty$.

Find the current I_2

(c) at $t = 0$,

(d) at $t = \infty$.

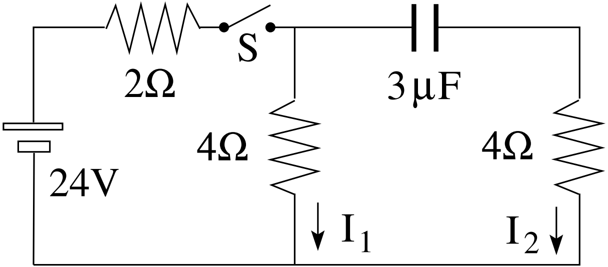


RC Circuit: Application (5)

In the RC circuit shown, the switch S has been open for a long time.

(a) Find the currents I_1 and I_2 immediately after the switch has been closed.

(b) Find the currents I_1 and I_2 a very long time later.



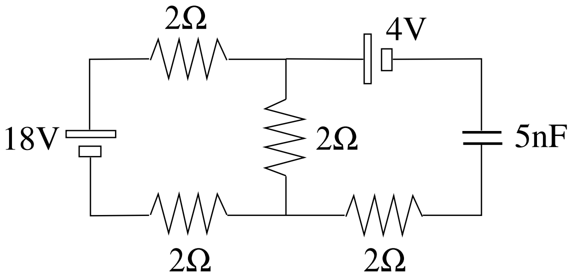
RC Circuit: Application (1)

This circuit has been running for a very long time.

(a) Find the current through the 18V battery.

(b) Find the total power dissipated in the resistors.

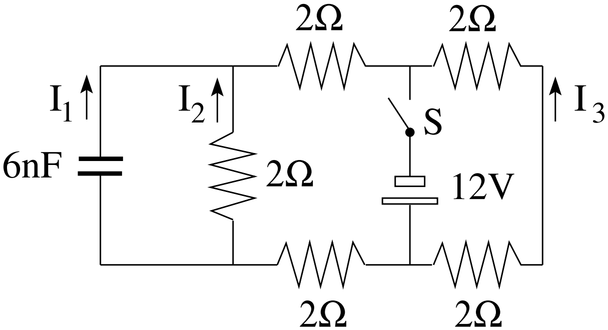
(c) Find the charge stored on the capacitor.



In this 3-loop RC circuit, the switch S is closed at time $t = 0$.

(a) Find the currents I_1, I_2, I_3 just after the switch has been closed.

(b) Find the currents I_1, I_2, I_3 a very long time later.



RC Circuit: Application (6)

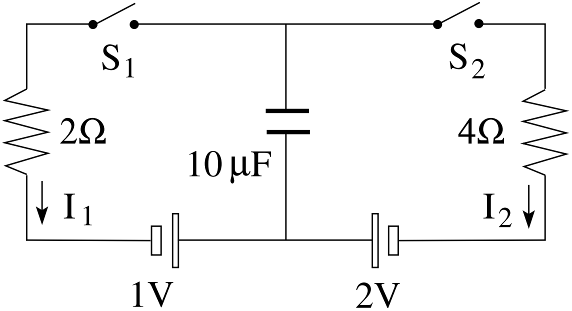
In the RC circuit shown, both switches are initially open and the capacitor is discharged.

(a) Close switch S_1 and find the currents I_1 and I_2 immediately afterwards.

(b) Find the currents $I_1 I_2$ and the charge Q on the capacitor a very long time later.

(c) Now close switch S_2 also and find the currents I_1 and I_2 immediately afterwards.

(d) Find the currents I_1, I_2 and the charge Q on the capacitor a very long time later.



RC Circuit: Application (2)

The switches are closed at $t = 0$. This begins the charging process in each RC circuit.

Name the circuit in which...

(i) the charge flows into the capacitor at the highest rate initially,
(ii) the capacitor has the most charge ultimately,
(iii) the capacitor is 63% full in the shortest time.